Revised Multilinear Regression Equations for Prediction of Lateral Spread Displacement

T. Leslie Youd[1]; Corbett M. Hansen[2]; and Steven F. Bartlett[3]

Abstract: In 1992 and 1995, Bartlett and Youd introduced empirical equations for the prediction of lateral spread displacement; these equations have gained wide use in engineering practice. The equations were developed from the multilinear regression (MLR) of a large case history database. This study corrects and updates the original analysis. Corrections and modifications include: (1) Bartlett and Youd erroneously overestimated measured displacements for lateral spreads generated by the 1983 Nihonkai-Chubu, Japan earthquake; those errors are corrected herein. (2) Several sites were deleted where boundary shear impeded free lateral displacement. (3) Data were added from three additional earthquakes. (4) The functional form of the mean-grain-size term was modified from ($D50_{15}$) to log($D50_{15}$ + 0.1 mm) to produce improved prediction of displacements for coarse-grained granular sites. (5) The functional form of the model was changed from log(R) to log(R^*), where R^* is a function of the magnitude of the earthquake, to prevent unrealistic overprediction of displacements when R becomes small. The revised data were re-regressed to generate new MLR equations. The new equations are recommended for engineering practice.

DOI: 10.1061/(ASCE)1090-0241(2002)128:12(1007)

CE Database keywords: Lateral displacement effect; Prediction; Data analysis.

Introduction

In the early 1990's, Bartlett and Youd (1992, 1995) introduced an empirical equation for predicting lateral spread displacement at liquefiable sites. Since that time, the equation has gained widespread use in engineering practice. The equation was developed through multilinear regression (MLR) of a large case history database compiled by these investigators. Over the years, several needed corrections and improvements to the equation have come to our attention. These corrections and improvements are addressed herein:

1. Bartlett and Youd, much to their chagrin, entered erroneous estimates of measured ground displacements into their database for lateral spreads generated by the 1983 Nihonkai-Chubu, Japan earthquake. The erroneous estimates were 1.9 times larger than the measured values reported by Hamada et al. (1986). This erroneous data led to a slight overprediction (about 8% on average) for ground slope failures compared with the revised model presented herein. This error is corrected in the reanalysis.
2. Bartlett and Youd incorporated data from several sites where boundary effects significantly impeded free lateral movement of the mobilized ground. To be consistent with a free lateral movement condition, eight displacement vectors (or sites) were removed form the database from localities where free lateral movement was clearly impeded. Removal of these data is conservative in that regression of the database with these sites removed leads to slightly greater predicted displacements compared to the previous regression.
3. Additional case history data were added from three earthquakes—1983 Borah Peak, Idaho, 1989 Loma Prieta, Calif., and 1995 Hyogo-Ken Nanbu (Kobe), Japan. The added sites contain data from several coarse-grained liquefiable sites, allowing the extension of the predictive equation to coarser-grained materials than is allowed by Bartlett and Youd (1992, 1995). The compiled case history data used in this investigation, including the newly added data, are listed on the senior writer's web site (Youd 2002).
4. The form of the equation was changed to incorporate the logarithm of the mean grain size term rather than the arithmetic term used by Bartlett and Youd. This change leads to greatly improved predictions for soils with mean grain sizes greater than 1.0 mm.
5. The form of the equation was changed from log(R) to log(R^*), where

$$R^* = R_o + R \tag{1}$$

and

$$R_o = 10^{(0.89M - 5.64)} \tag{2}$$

R = the horizontal or mapped distance from the site in question to the nearest bound of the seismic energy source, R_o = a distance constant that is a function of earthquake magnitude, M, and R^* = the modified source distance. This modification prevents the prediction of unrealistically large displacements when R becomes small, a shortcoming of the Bartlett and Youd equation.

These modifications were applied in a stepwise MLR procedure. The result of each regression is tabulated and the derived coefficients compared with those of the previous step to assure

[1]Professor, Civil Engineering Dept., Brigham Young Univ., Provo, UT 84602-4081.

[2]Project Engineer, Kleinfelder, Inc., Salt Lake City, UT 84109-1617.

[3]Assistant Professor, Civil Engineering Dept., Univ. of Utah, Salt Lake City, UT 84112.

Note. Discussion open until May 1, 2003. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on June 1, 2001; approved on February 21, 2002. This paper is part of the ***Journal of Geotechnical and Geoenvironmental Engineering***, Vol. 128, No. 12, December 1, 2002. ©ASCE, ISSN 1090-0241/2002/12-1007–1017/$8.00+$.50 per page.

Table 1. Multilinear Regression Coefficients Determined by Bartlett and Youd (1992, 1995)

Parameters	Constants		M	Log R	R	Log W	Log S	Log T_{15}	Log (100-F_{15})	$D50_{15}$	Regression coefficient
Coefficients	b_o	b_{off}	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	R_c^2
Values	−15.787	−0.579	1.178	−0.927	−0.013	0.657	0.429	0.348	4.527	−0.922	82.6

that outcomes are reasonable and logical. The values of the coefficient of determination, R_c^2, were also compared to evaluate the overall performance of the model. The analyses described herein begin with the Bartlett and Youd (1992, 1995) equation and end at Step 5 with a new MLR model. Eqs. (6*a*) and (6*b*) are now recommended by the authors for use in engineering practice for prediction of lateral spread displacement.

An interim report produced by the authors (Youd et al. 1999) introduced many of the changes and modifications developed in this study. The present paper incorporates additional analyses and revisions and the final equations supersede those in the 1999 interim report.

Bartlett and Youd Equation

The general form of the Bartlett and Youd (1992, 1995) equation is:

$$\text{Log } D_H = b_o + b_{\text{off}} + b_1 M + b_2 \text{ Log } R + b_3 R + b_4 \text{ Log } W + b_5 \text{ Log } S + b_6 \text{ Log } T_{15} + b_7 \text{ Log}(100 - F_{15}) + b_8 D50_{15} \quad (3)$$

D_H = the estimated lateral ground displacement, in meters; M = the moment magnitude of the earthquake, R = the nearest horizontal or map distance from the site to the seismic energy source, in kilometers, T_{15} = the cumulative thickness of saturated granular layers with corrected blow counts, $(N_1)_{60}$, less than 15, in meters, F_{15} = the average fines content (fraction of sediment sample passing a No. 200 sieve) for granular materials included within T_{15}, in percent, $D50_{15}$ = the average mean grain size for granular materials within T_{15}, in millimeters, S = the ground slope, in percent, and W = the free-face ratio defined as the height (H) of the free face divided by the distance (L) from the base of the free face to the point in question, in percent. The various b values (Table 1) are regression coefficients derived from an MLR analysis. The coefficient b_o is a general intercept for the complete MLR equation and the coefficient b_{off} is an intercept adjustment that is added for free-face conditions. Regression coefficients for the Bartlett and Youd equation are listed in Table 1. These coefficients are significant at the 99.9% confidence level and the correlation coefficient, R_c^2, for the model is 82.6%.

The general predictive capability of the Bartlett and Youd equation is demonstrated in Figs. 1 and 2, where measured displacements plotted against predicted displacements from Eq. (3) and the coefficients listed in Table 1. Displacements up to 15 m in magnitude are plotted in Fig. 1 and displacements up to 2 m, which are of greater interest to engineers, are plotted in Fig. 2. These plots indicate that Eq. (3) is valid for predicting lateral spread displacements within a factor of about two for sites characterized by soil and other properties within the limits of the compiled database and the constraints discussed by Bartlett and Youd (1992, 1995).

Modifications to Bartlett and Youd Equation

Step 1—Correction of Miscalculated Nihonkai-Chubu, Japan Displacements

The first modification to the Bartlett and Youd model was the correction of miscalculated displacements from the 1983 Nihonkai-Chubu, Japan earthquake. That correction was made by dividing the magnitude of each miscalculated displacement by a factor of 1.9 to yield correct values. With this correction made, the regression was redone using the revised data and the functional form used by Bartlett and Youd (1992, 1995). This regres-

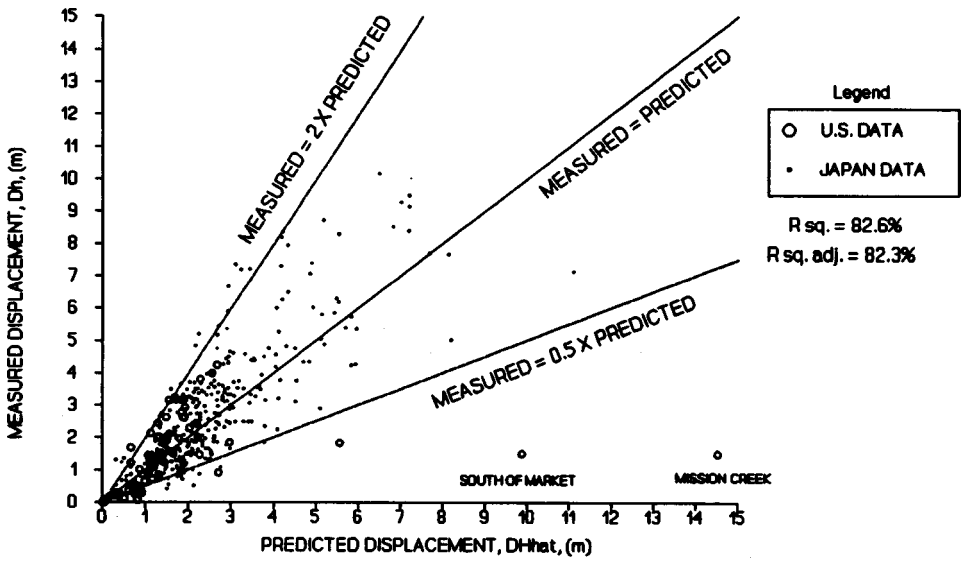

Fig. 1. Measured versus predicted displacement using Bartlett and Youd Model (Bartlett and Youd 1992)

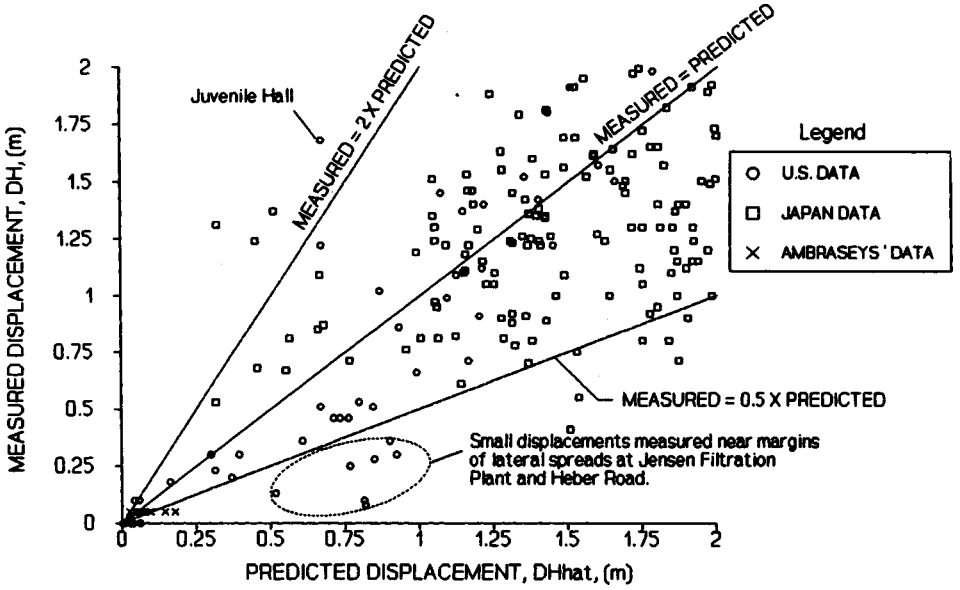

Fig. 2. Measured versus predicted displacements for displacements up to 2 meters using Bartlett and Youd Model (Bartlett and Youd 1992)

sion produced the set of regression coefficients listed in Table 2. All coefficients are significant at the 99.9% confidence level and the R^2 for the regression is 81.0%, which is slightly lower than the 82.6% calculated from the Bartlett and Youd equations.

The percentage change for each coefficient, compared to the Bartlett and Youd coefficients, is also listed in Table 2. For example, b_5, the coefficient for the slope parameter, S, decreased by 35.9%. This change was expected because all of the corrected displacements were on sloping ground. Surprisingly, a larger change (plus 42.0%) occurred in b_6, the coefficient for the thickness parameter, T_{15}. This large change apparently occurred because the miscalculated displacements were also at sites with relatively thin, 2–3 m thick, T_{15} layers. The erroneously large displacements at sites with relatively thin T_{15} layers apparently caused the Bartlett and Youd regression to underestimate the importance of the thickness term, T_{15}. Thus, the corrections in Step 1 lead to generally smaller predicted displacements for sites on sloping ground (about 8% smaller, on average), but somewhat larger predictions for sites with thick T_{15} layers.

Step 2—Removal of Sites where Boundary Effects Impeded Displacement

The second modification to the MLR model was the removal of eight displacement vectors for sites where lateral spread displacements were clearly impeded by shear or compression forces along the margins or at the toe of the lateral spread. Because of boundary effects, predicted displacements for these sites are several times greater than the corresponding measured displacements. Also, removal of these data is conservative in that their removal leads to slightly greater predicted displacements than if the eight data were included. Because our objective is to develop conservative equations that are valid for freely moving lateral spreads, these questionable data were removed.

Two of the removed sites are marked in Fig. 1 as ''Mission Creek'' and ''South of Market.'' Both of those lateral spreads occurred during the 1906 San Francisco earthquake. Changes of direction within the Mission Creek zone and buttressing at the toe of the South of Market zone apparently impeded displacement of these failures (Youd and Hoose 1978). These sites are both relatively close to the seismic energy source for the large (magnitude 8.1) causative earthquake, which may be another factor leading to the relatively large predicted displacements. The other sites removed were six displacement vectors noted in Fig. 2 as ''margin of lateral spreads at Jensen Filtration Plant and Heber Road.'' Boundary shear and other effects apparently prevented free movement along the margins of these spreads. Displacements in the interior of these spreads were apparently unaffected by boundary shear; data from those sites were left in the database.

After removal of data from the eight affected sites, the database was regressed generating the coefficients listed in Table 3. All coefficients are significant at the 99.9% confidence level and the R_c^2 increased to 84.5%. The only significant changes produced by the removal of the eight sites were decreases of 11.3% and 7.1% percent in coefficients b_2 and b_3, respectively. These coef-

Table 2. Coefficients Regressed from Step 1—Correction of Miscalculated Displacement from 1983 Nihonkai-Chubu, Japan Earthquake

Parameters	Constants		M	Log R	R	Log W	Log S	Log T_{15}	Log (100-F_{15})	$D50_{15}$	Regression coefficient
Coefficients	b_o	b_{off}	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	R_c^2
Values	−14.55	−0.483	1.096	−.0873	−0.014	0.634	0.275	0.494	4.053	−0.814	81.0
Change from Table 1	1.236	0.096	−0.082	0.054	−0.001	−0.023	−0.154	0.146	−0.474	0.108	−1.6
% Chance from Table 1	7.8%	16.6%	−7.0%	5.8%	−7.7%	−3.5%	−35.9%	42.0%	−10.5%	11.7%	−1.9%

Table 3. Coefficients Regressed from Step 2—Data Removed for Sites Where Boundary Effects Retarded Displacement

Variables	Constants		M	Log R	R	Log W	Log S	Log T_{15}	Log (100-F_{15})	$D50_{15}$	Regression coefficient
Coefficients	b_o	b_{off}	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	R_c^2
Values	−14.68	−0.449	1.14	−0.972	−0.013	0.591	0.281	0.507	4.012	−0.867	84.5
Change from Table 2	−0.124	0.034	0.044	−0.099	0.001	−0.043	0.006	0.013	−0.041	−0.053	3.5
% Change from Table 2	−0.9%	7.0%	4.0%	−11.3%	7.1%	−6.8%	2.2%	2.6%	−1.0%	−6.5%	4.3%
Change from Table 1	1.112	0.13	−0.038	−0.045	0	−0.066	−0.148	0.159	−0.515	0.055	1.9
% Change from Table 1	7.0%	22.5%	−3.2%	−4.9%	0.0%	−10.0%	−34.5%	45.7%	−11.4%	6.0%	2.3%

ficients modify the distance terms, $\log R$ and R. These changes cause small increases of predicted displacements near the seismic energy source.

Step 3—1983 Borah Peak, Idaho; 1989 Loma Prieta, California; and 1995 Kobe, Japan Earthquakes

Several new sites were added to the database, including three gravely sites from the Whisky Springs lateral spread (1983 Borah Peak, Idaho earthquake), two sites from a lateral spread at the Monterey Bay Aquarium Research Institute (MBARI), Moss Landing, Calif. (1989 Loma Prieta earthquake), and 19 sites from lateral spreads generated by the 1995 Hyogo-Ken Nanbu (Kobe) Japan, earthquake. Many of the added sites are underlain by coarse sands and gravels, some of which (Whiskey Springs) had been removed by Bartlett and Youd (1992, 1995) from their database because they were deemed too coarse grained (mean grain size greater than 1.0 mm) to be accurately predicted by their model.

1983 Borah Peak Earthquake

The Whisky Springs spread was caused by liquefaction of well-graded sediment composed of silt, sand, and gravel during the 1983 Borah Peak earthquake; (Youd et al. 1985; Andrus and Youd 1987). Three displacement vectors, each with a lateral displacement of about 1.0 m, were added to the database corresponding to three boreholes drilled through the body of the spread. Average mean grain sizes, $D50_{15}$, in the T_{15} layers identified within these holes ranged from 3.0 to 10 mm and average fines contents ranged from 15 to 30%.

Because of the relatively high fines contents, these gravelly materials have permeabilities roughly equivalent to silty sand. Slow drainage, in this instance caused by low permeability, appears to be an important factor controlling the amount of lateral spread movement. The relatively low permeability prevents rapid dissipation of excess pore pressures, allowing the liquefied layer to remain mobile throughout the time of strong ground shaking and beyond, and allowing large displacement to accumulate.

Data from a nearby clean gravel site, named Pence Ranch, were not added to the database for this analysis for the following reasons. This site liquefied and lateral spread displacement occurred over part of the site during the 1983 earthquake. Where lateral spread occurred, the measured displacements were well predicted by the equations developed in this paper. However, the soil conditions were similar to other localities where lateral spread did not occur, except that a silt layer capping the gravel was missing in areas without lateral spread displacement (Andrus et al. 1991). In the latter instance, rapid drainage for the gravel layer apparently inhibited lateral spread. Because predicted displacements are approximately the same for the two areas, an additional factor, hydraulic conductivity of the gravel and overlying layers, appears to be an important factor not accounted for in the MLR equations of Bartlett and Youd. Not enough case history data are available in the compiled information to adequately account for the influence of hydraulic conductivity of coarse-grained materials. As discussed later, the final model developed herein is restricted to granular soils with sufficient fine sand and silt to impeded rapid dissipation of excess pore water pressures during the interval of strong earthquake shaking.

1989 Loma Prieta Earthquake

During the 1989 Loma Prieta, earthquake, several lateral spreads developed in and near Moss Landing, Calif. (Mejia 1998). Displacement of a lateral spread along Sandholdt Road was accurately determined from deformation of slope inclinometer casings placed prior to the earthquake. Displacements of 74 and 280 mm, respectively, from these two inclinometer sites were added to the database. These sites are underlain by liquefiable soils composed of medium to coarse sands with up to 8% fines (Boulanger et al. 1995).

1995 Hyogo-Ken Nanbu (Kobe), Japan Earthquake

During the 1995 Hyogo-Ken Nanbu (Kobe), Japan earthquake, lateral spread occurred pervasively within and around the margins of Port and Rokko Islands and at several filled areas along the shoreline of Osaka Bay. Hamada et al. (1995) used pre- and postearthquake aerial photographs to calculate displacement vectors for these areas. Those vectors indicate that lateral spreads pushed perimeter quay walls from 2 to 6 m laterally toward the adjacent bay. The walls apparently provided little resistance to the lateral spread movements. Inland from the walls, displacement vectors were generally directed toward the nearest wall, but systematically decreased with distance from the wall. At distances between 50 and 300 m from the walls, displacements generally ranged between 2 and 0.1 m. Beyond 300 m, the mapped displacements were generally small and chaotic in orientation, indicating that a consistent pattern of displacement did not develop or that the magnitude of displacements were near the detection limit of the photogrammetric technique.

Numerous borehole logs with standard penetration test (SPT) data are reported by Hamada et al. (1995) for island areas, but no grain-size data are contained on these logs. The logs allowed accurate calculation of T_{15} at many localities, including localities near mapped displacement vectors. These T_{15} values, however, were rather uniform in thickness, ranging from 11 to 13 m on Port Island and 15 to 17 m on Rokko Island.

Only limited grain-size data were available from the Kobe area at the time of our study. We collected data from three logs from Port Island (Fig. 3), two logs from Rokko Island, and one log form an area referenced as LP gas tank yard, located on the mainland directly across the ship channel from Rokko Island. These unpublished logs, supplied by Kobe City and the Port and Harbors Research Institute, contain SPT N values, mean-grain size and fines-content information at 1 m sampling intervals. These

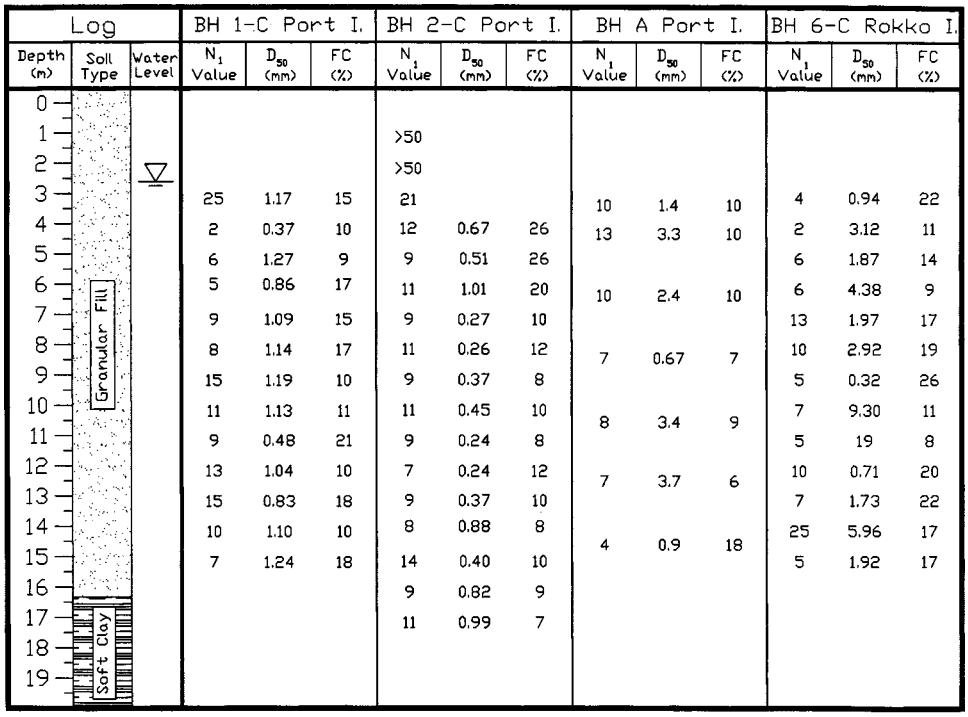

Fig. 3. Borehole logs from Port and Rokko Islands and LP gas site where lateral spread occurred during the 1985 Hyogo-Ken Nambu (Kobe) Earthquake (data courtesy of Kobe City and the Port and Harbors Research Institute, Japan).

data indicate that the fills are very heterogeneous, with widely varying grain-size distributions over short distances, both horizontally and vertically. Even within a single borehole, mean-grain size may vary by factors of three or more between adjacent samples. Because of the large variation of $D50$ values, $D50_{15}$ was determined for each log by calculating the geometric mean of reported mean-grain sizes within the interval characterized by T_{15}. For these sites, the geometric means was used, rather than the arithmetic average as prescribed by Bartlett and Youd (1992), because the geometric mean is less influenced by extreme values in a heterogeneous data set. Thus, we recommend the use of the geometric mean to calculate $D50_{15}$ values for sites where the maximum value of $D50$ in a data set is greater than three times the minimum value. Although there were significant variations in fines content as well, these variations were less than a factor of three between minimal and maximum values and the arithmetic average for F_{15} was used as specified by Bartlett and Youd (1992).

To add site data and displacement vectors from the Kobe area, zones 100 m wide and 350 m long and oriented perpendicular to the nearest wall, were drawn centered on the borehole logs with available grain-size data, as noted in Figs. 3(a–c). One log on Rokko Island was not used because the borehole was located near the center of the island where lateral spread did not occur. Also, displacement vectors located directly on or within 50 m of a quay wall were not used because those displacements may have been influenced by soil–structure interaction or flow failure. Using these criteria, 19 displacement vectors were added to the database, including 8 vectors from Port Island; 6 vectors from Rokko Island; and 5 vectors from the LP gas yard.

Analysis

The expanded database was regressed, producing the coefficients listed in Table 4. These coefficients are significant at the 99.9% confidence level. The addition of this data, however, leads to a model with a significantly smaller correlation coefficient (80.6% compared to the previous 84.5%) and large under prediction of displacements for sites characterized by mean-grain sizes greater than 1.0 mm, the maximum allowed for the Bartlett and Youd equation. The poor performance of this model necessitates a change of functional form (addressed in Step 4) to improve the predictive performance of the model.

Table 4. Coefficients Regressed from Step 3—Data Added from Recent Earthquakes

Variables	Constants		M	Log R	R	Log W	Log S	Log T_{15}	Log $(100\text{-}F_{15})$	$D50_{15}$	Regression coefficient
Coefficients	b_o	b_{off}	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	R_c^2
Values	−12.958	−0.393	1.077	−0.902	−0.015	0.488	0.320	0.475	3.255	−0.167	80.3
Change from Table 3	1.722	0.063	−0.063	0.070	−0.002	−0.103	0.039	−0.032	−0.757	0.700	−4.2
% Change from Table 3	11.7%	12.5%	−5.5%	7.2%	−15.4%	−17.4%	13.9%	−6.3%	−18.9%	80.7%	−5.0%
Change from Table 1	2.829	0.186	−0.101	0.025	−0.002	−0.169	−0.109	0.127	−1.272	0.755	−2.3
% Change from Table 1	17.9%	32.1%	−8.6%	2.7%	−15.4%	−25.7%	−25.4%	36.5%	−28.1%	81.9%	−2.8%

Table 5. Coefficients Regressed from Step 4—Change of Functional Form to Log $b_8^*(D50_{15}+0.1$ millimeter)

Variables	Constants		M	Log R	R	Log W	Log S	Log T_{15}	Log (100-F_{15})	$D50_{15}$	Regression coefficient
Coefficients	b_o	b_{off}	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	R_c^2
Values	−15.356	−0.456	1.100	−0.911	−0.014	0.554	0.325	0.546	4.168	−0.852	84.2
Change from Table 5	−2.398	−0.063	−0.023	−0.009	0.001	0.066	0.005	0.071	0.913	−0.685	3.9
% Change from Table 5	−18.5%	−16.0%	2.1%	−1.0%	6.7%	13.5%	1.6%	14.9%	28.0%	−410.2	4.9
Change from Table 1	0.431	0.123	−0.078	0.016	−0.001	−0.103	−0.104	0.198	−0.359	0.070	1.6
% Change from Table 1	2.7%	21.2%	−6.6%	1.7%	−7.7%	−15.7%	−24.2%	56.9%	−7.9%	7.6	1.9

Step 4—Change of Form from $D50_{15}$ to log($D50_{15}$ + 0.1 mm)

The grain-size factor, $b_8 D50_{15}$, as incorporated in the Bartlett and Youd model and the model in Step 3, causes the predictive equation to be very sensitive to mean-grain size. To improve the predictive performance of the model and to reduce sensitivity, the functional form of the equation was changed from $b_8 D50_{15}$ to $b_8 \log(D50_{15}+0.1\text{ mm})$. (The 0.1 mm value was added to $D50_{15}$ to prevent the prediction of unrealistically large values of displacement should values of $D50_{15}$, approaching zero be inadvertently entered into the equation.) With this change of form, the MLR analysis yields the coefficients listed in Table 5. All coefficients are significant at the 99.9% confidence level and the correlation coefficient, R_c^2 improved to 84.1%.

The change in form of the MLR model caused an expected large reduction (from −0.166 to −0.819) in b_8, the coefficient for $D50_{15}$, and a correspondingly large increase (from 3.266 to 4.130) in b_7, the coefficient for F_{15}. The correspondingly large adjustments to both b_7 and b_8 occur because of an interrelationship between mean-grain size and fines content for typical soils. The remaining coefficients were not greatly affected or returned to values generally within 10% of those regressed in Step 2. Thus, the change of model form restored the various coefficients to values with about the same relative influence on predicted displacements as occurred in Step 2.

Step 5—Change of Functional Form from log(R) to log(R)*

The functional form of the $b_2 \log(R)$ term in the Bartlett and Youd (1992, 1995) model leads to the calculation of unrealistically large displacements as R approaches zero. To mitigate this problem, Bartlett and Youd (1992, 1995) specified a set of minimal R values. These distances are a function of magnitude and vary from 0.5 to 30 km. To eliminate the need for these prescribed minimum R values, we added a magnitude-dependent factor to the log(R) term. This factor prevents calculation of excessively large displacements at small source distances and eliminates the need for the minimum R values specified by Bartlett and Youd (1992, 1995).

The adjustment to R is made by changing the form of the equation from log(R) to log(R^*), where

$$R^* = R + R_o \tag{4}$$

and

$$R_o = 10^{(0.89M - 5.64)} \tag{5}$$

R = the horizontal or mapped distance from the site in question to the nearest bound of the seismic energy source, R_o = a distance factor that is a function of earthquake magnitude, M, and R^* = a modified source distance value. Note that the modified source distance, R^*, applies only to the log term, $b_2 \log(R^*)$ in the equation. The measured distance R is used in the arithmetic term, $b_3 R$. Although addition of R^* to the model eliminates the need for the set of minimal R values specified by Bartlett and Youd, R values approaching zero yields large and somewhat uncertain results. Because there are few displacements greater than 6 m in the case history data, the accuracy of predicted displacements greater than 6 m is questionable. Also, the MLR equation may yield erratic results for very small source distances. Because of this problem, R values smaller than 0.5 km should not be applied. (For source distances less than 0.5 km, a value of 0.5 km should be used.)

The dataset was reregressed with ($\log R$) replaced by ($\log R^*$), yielding the coefficients listed in Table 6. All coefficients are significant at the 99.9% confidence level. R_c^2 decreased slightly from 84.1% (Step 4) to 83.6%. The change of the MLR model with the added a correlation between M, R, and R^*, induced large changes to b_1, the coefficient for M, and b_2, the coefficient for R^*. The net result is smaller predicted displacements near the seismic source zone, but very little change in predicted displacements beyond the near-source zone.

With these modifications to the original Bartlett and Youd model, the coefficients listed in Table 6 provide the final revised MLR model that we recommend for engineering use. Eq. (6*a*) gives the MLR model for free-face conditions:

$$\begin{aligned}\log D_H = &-16.713 + 1.532M - 1.406 \log R^* - 0.012R \\ &+ 0.592 \log W + 0.540 \log T_{15} + 3.413 \log(100 - F_{15}) \\ &- 0.795 \log(D50_{15} + 0.1\ \text{mm})\end{aligned} \tag{6a}$$

Table 6. Coefficients Regressed from Step 5—Change of Form from Log R to Log R^*

Variables	Constants		M	Log R^*	R	Log W	Log S	Log T_{15}	Log (100-F_{15})	$D50_{15}$	Regression coefficient
Coefficients	b_o	b_{off}	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	R_c^2
Values	−16.213	−0.500	1.532	−1.406	−0.012	0.592	0.338	0.540	3.413	−0.795	83.6
Change from Table 8	−0.857	−0.044	0.432	−0.495	0.002	0.038	0.013	−0.006	−0.755	0.057	−0.6
% Change from Table 8	−5.6	−9.6	39.3	−54.3	14.3	6.9	4.0	−1.1%	−18.1%	6.7	−0.7
Change from Table 1	−0.426	0.079	0.354	−0.479	0.001	−0.065	−0.091	0.192	−1.114	−0.127	1.00
% Change from Table 1	−2.7	13.6	30.1	−51.7	7.7	−9.9	−22.2	55.2	−24.6	−13.8	1.2%

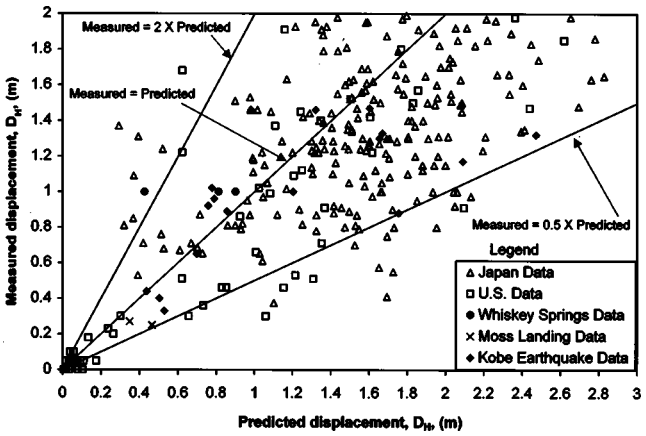

Fig. 4. Measured versus predicted displacements using new model [Eq. (6)] for displacements up to 2 meters

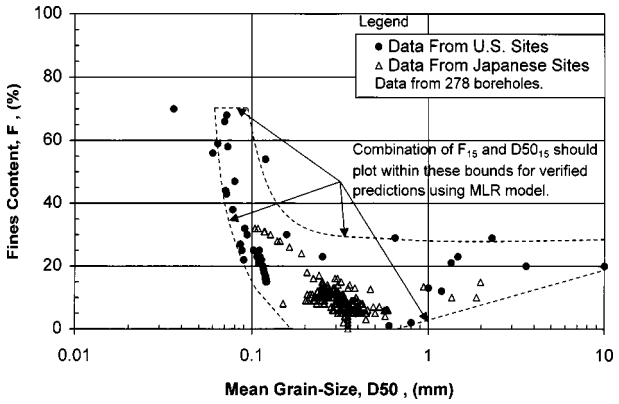

Fig. 5. Compiled grain-size data with ranges of F_{15} and $D50_{15}$ [for which Eq. (6) is applicable]

Eq. (6*b*) is applied to gently sloping ground conditions:

$$\log D_H = -16.213 + 1.532M - 1.406 \log R^* - 0.012R + 0.338 \log S + 0.540 \log T_{15} + 3.413 \log(100 - F_{15}) - 0.795 \log(D50_{15} + 0.1 \text{ mm}) \quad (6b)$$

R^* and R_o are defined by Eqs. (4) and (5).

Performance and Limitations of Revised Multilinear Regression Equations

The predictive capability of Eqs. (6*a*) and (6*b*) is illustrated by the comparative data plotted in Fig. 4. On that plot, measured displacements from the database are plotted against predicted displacements for displacements up to 2 m. The great majority of predicted displacements plot within a factor of two of the measured values. Although better predictive capability is desirable, the achieved level is near the maximum that the quality of the case history data will allow. This level of accuracy is consistent with the original Bartlett and Youd equations and is adequate for most routine engineering applications.

Coarse-Grained Sediment

A major improvement generated by this study is the extension of the range of mean-grain sizes and fines contents for which the MLR equations can be applied. Fig. 5 is a plot of average fines content, F_{15}, versus average mean-grain size, $D50_{15}$, for sites in the database. Bounds are also marked on the plot outlining the region for which the data provide sufficient constraint for application of Eqs. (6*a*) and (6*b*). This plot indicates that $D50_{15}$ values as large as 10 mm and F_{15} values as great as 70% are sufficiently represented in the data base to allow the use of the MLR within the defined limits (Fig. 5).

To demonstrate the improved performance of the revised equations for $D50_{15}$ greater than 1 mm, predicted versus measured displacements, using both Eqs. (6*a*) and (6*b*) and the Bartlett and Youd equation, for these larger grain sizes are plotted in Fig. 6. The severe underprediction of displacements using the Bartlett and Youd equations (generally less than one tenth the measured values) clearly demonstrates the inadequacy of the Bartlett and Youd equations for predicting displacements for coarse-grained sites. On the other hand, predictions from Eqs. (6*a*) and (6*b*) are generally with in a factor of two of the measured values, indicating that these equations are sufficiently robust to extend the MLR model to $D50_{15}$ values as large as 10 mm.

One limitation to the use of Eqs. (6*a*) and (6*b*) for coarse-grained sites is that the equations are only valid for soils with impeded drainage. All of the gravels included in the database have sufficient fines content, as noted in Fig. 5, to yield hydraulic conductivities equivalent to sands or silty sands. These low hydraulic conductivities impeded drainage during the interval of strong ground shaking and allow full development of lateral spread displacement to occur. Where drainage can occur rapidly, Eqs. (6*a*) and (6*b*) may over predict displacement. For example, two sets of predicted and measured displacements are plotted in Fig. 6 for the Pence Ranch site where liquefaction occurred during the 1983 Borah Peak, Idaho earthquake. The Pence Ranch data were not included in the data set regressed for this study. One predicted displacement (0.18 m) is within a factor of two of the 30 mm of measured movement at this site. Displacement at the second locality a few meters away, however, was predicted at 0.23 m while the measured displacement was zero. The soil properties in the liquefiable layer at both sites are similar (Andrus et al. 1991). The locality with 0.18 m of displacement is capped by a low-permeability silty layer that apparently impeded drainage. That capping layer was absent at the locality where 0.23 m of displacement is predicted, but none occurred.

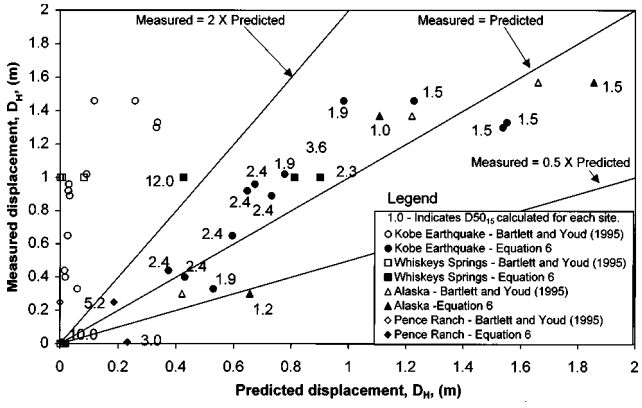

Fig. 6. Measured versus predicted displacements for sites with $D50_{15}$ equal to or greater than 1 millimeter; displacements are predicted within a factor of 2 [by Eq. (6)], but are greatly underpredicted by the Bartlett and Youd (1992, 1995) equation

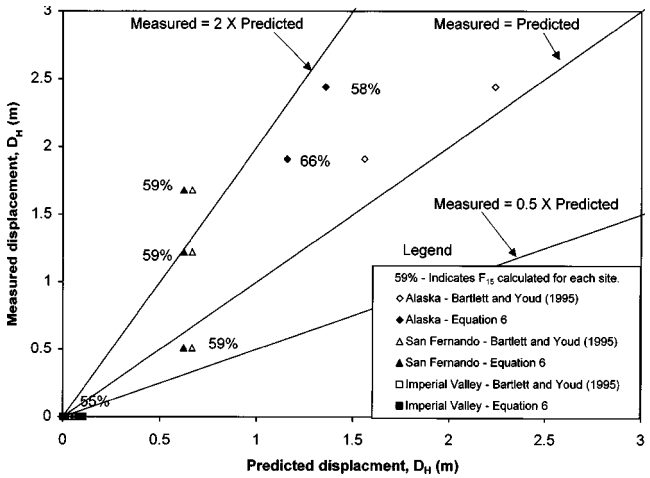

Fig. 7. Measured versus predicted displacements for sites with F_{15} greater than 50%; measured displacements are predicted within a factor of 2 for sediments with fine contents as great as 70%

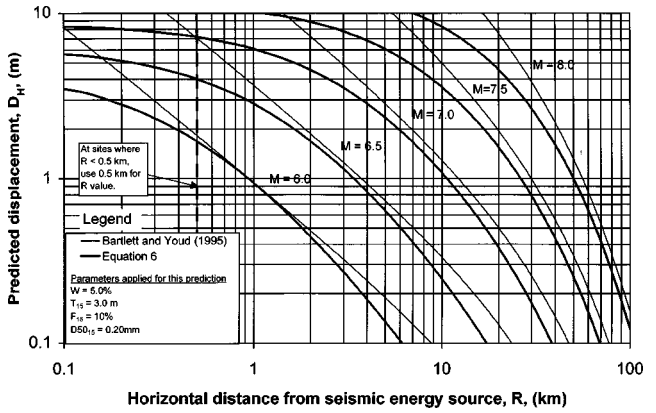

Fig. 8. Predicted displacements as a function of source distance, *R*, for typical free-face site conditions [using both Eq. (6) and the Bartlett and Youd (1992, 1995) equation]

Several other localities at which lateral spread did not occur in clean, free draining gravelly materials have been reported in literature. For example, Youd et al. (1985) noted a lack of either surficial liquefaction effects or lateral spread in gravelly sediments along an 8 km long reach of the shallowly incised Lost River in the heavily shaken epicentral region of 1983 the Borah Peak earthquake. These gravels were late Holocene in age and in some localities appeared clean and loose. Presumably rapid drainage prevented liquefaction and lateral spread from occurring in these deposits. Youd (1977) also noted a similar lack of evidence of liquefaction or lateral spread in active floodplains underlain by cobbles and coarse gravel in the epicentral area of a magnitude 7.2 earthquake in Romania in 1977. About 1.0 km of the active floodplain of the Putna River was searched for evidence of liquefaction and lateral spread after that earthquake, with no effects observed.

Fine-Grained Sediment

Fig. 7 is a plot of predicted versus measured displacements for sites characterized by F_{15} equal to or greater than 50%. These displacements are predicted within a factor of two, indicating that Eq. (6) can be applied at silty sites with fines contents as high as 70% so long as the fines are nonplastic. Highly plastic fines are generally nonliquefiable and hence not susceptible to lateral spread.

Near-Fault Localities

The change of form from $\log R$ to $\log R^*$ introduced in Step 5 reduces excessively large displacements predicted by the Bartlett and Youd equation at localities near the seismic energy source. (The reason that the Bartlett and Youd equation predicts very large displacements near the seismic energy source is because values of $\log R$ approach negative infinity as R approaches zero.) To prevent the distance term from approaching zero, we modified R to R^* [Eq. (1)], where R^* includes the magnitude dependent factor R_o [Eq. (5)]. The result of this modification is illustrated in Fig. 8 where predicted displacements from both the Bartlett and Youd equations and Eq. (6) are plotted versus R for a typical site condition and several earthquake magnitudes. Fig. 8 shows a large reduction in predicted displacements at small source distances using Eq. (6*a*) relative to the Bartlett and Youd model. Unfortunately, there are not many lateral spread case histories in the dataset for near the seismic source. Thus, we were unable to fully verify the accuracy of Eqs. (6*a*) and (6*b*) for small values of R. The few measured displacements at sites within 1 km of the fault (e.g., data for the 1971 San Fernando and 1979 Imperial Valley, California earthquakes) however, are predicted within a factor of two using the Eq. (6).

Despite the change of form from R to R^*, large displacements may still be calculated for large earthquakes near the seismic source. Upon further evaluation of the predictive limits of Eqs. (6*a*) and (6*b*), we suggest that any predicted displacement greater than 6 m is beyond the predictive range of the MLR model. Thus, predicted displacements greater than 6 m should be taken as an indicator that large displacements are possible, but the amount of predicted displacement is uncertain. Displacements larger than 6 m are not well constrained by the case history data. The only displacements larger than 6 m in the compiled database are from the banks of the Shinano River in Niigata, Japan where lateral spread occurred during the 1964 earthquake. Those banks displaced as much as 10 m toward the incised river channel (Fig. 1). Because only one earthquake and one site condition is embodied in these larger displacements, predicted displacements greater than 6 m are not well constrained by the case history database. When displacements larger than 6 m are predicted, they should not be relied upon as accurate predictions, but as indicators that large displacements are possible.

Comparison with Previous Models

Several comparisons between predicted lateral spread displacements from Eqs. (6*a* and *b*) and the Bartlett and Youd equations are given in the previous paragraphs and in Figs. 6–8. These comparisons indicate that Eq. (6) generally yields smaller predicted lateral spread displacements (about 8% on average) than the Bartlett and Youd equation, except for localities underlain by liquefiable sediment with large mean-grain sizes (generally $D50_{15}>1$ mm), high fines contents (generally $F_{15}>40\%$), or thick liquefiable layers (generally $T_{15}>4$ m). The larger predictions are more pronounced for earthquakes with magnitudes less than 6.5 and at localities near the seismic energy source (generally, $R<5$ km).

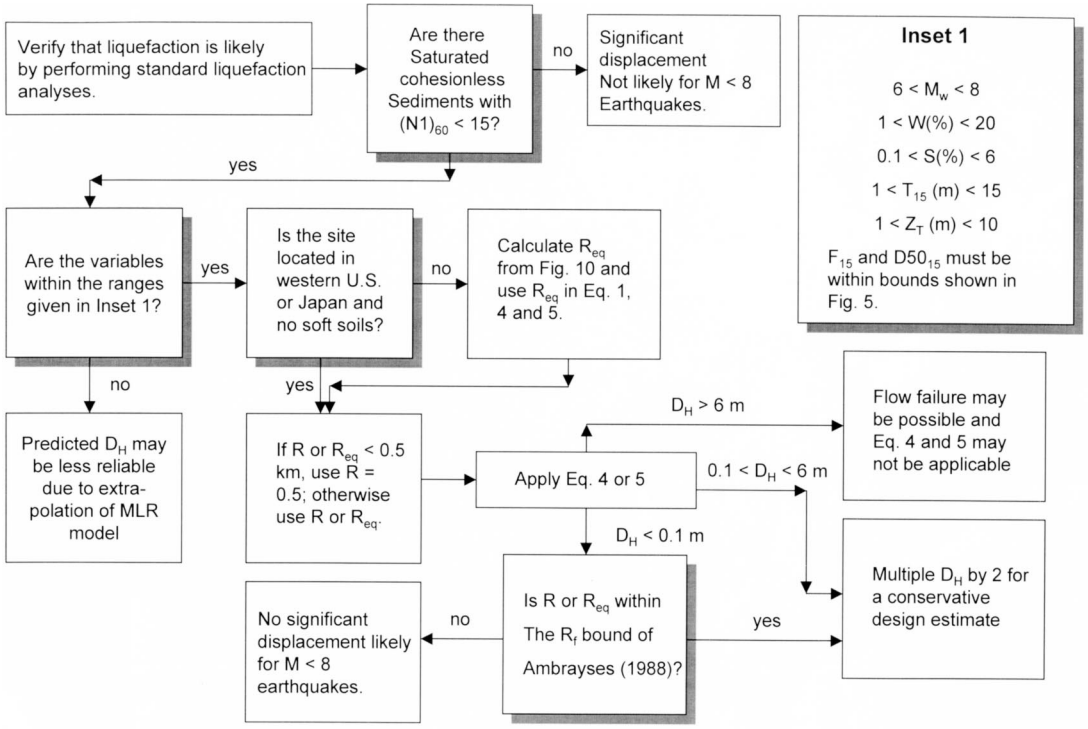

Fig. 9. Flow chart [for application of Eq. (6)]

One reason for generally slightly smaller predicted displacements is corrections of the erroneous data in Step 1. This correction led to a significant increase in coefficient b_6, the coefficient for the thickness term (T_{15}), which leads to the tendency for greater predicted displacements for sites underlain by thick liquefiable layers.

The change of functional form for the model in Step 4, from $b_8 D50_{15}$ to $b_8 \log(D50_{15}+0.1\text{ mm})$ leads to greater and more reliable predicted displacements for coarser-grained sediment. Also, the modifications introduced into the model led to slightly greater predicted displacements for sediments with a high fines content. In a preliminary report, the authors (Youd et al. 1999) recommended a set of interim equations for use in engineering practice. Eqs. (6*a*) and (6*b*) supersede those preliminary equations. Those preliminary equations yield predicted displacements that are generally slightly greater than those estimated from Eq. (6), except for very coarse-grained sediments ($D50_{15}>1$ mm) and the very fine-grained sediments ($F_{15}>40\%$). For these materials, the revised equations in this study predict slightly greater displacements. The equations presented here provide better results, as indicated by the increased R^2.

In the interim study, Youd et al. (1999) made several additional modifications were made to the MLR model, including the addition of a large number of sites from Port and Rokko islands where lateral spread was pervasive but where local grain-size data were not available. Average grain-size data was applied to make up for this deficiency. In another step, fines content was also arbitrarily capped at F_{15} of 55% as a measure of conservatism. Upon reexamination, those steps seemed somewhat arbitrary. Thus, those steps are omitted in this paper with the recommendation that the equations in the interim report be discontinued for use in engineering practice.

Hamada et al. (1986) compiled lateral spread and borehole data from Niigata and Noshiro, Japan and developed the following preliminary empirical equation for estimating lateral spread displacement

$$D=0.75H^{1/2}\theta^{1/3} \tag{7}$$

D = predicted lateral displacement, H = the thickness of the liquefied layer [roughly equivalent to T_{15} in Eqs. (6*a* and *b*)] and θ is ground slope [roughly equivalent to S in Eq. (6)]. The similarity of the exponential coefficients of 0.500 and 0.333 for H and θ, respectively, in Eq. (7), and 0.542 and 0.334 for T_{15} and S, respectively, in Eq. (6), may be more than coincidental. Although both equations are empirical, in that they were derived directly form compiled case history data, the underlying physical phenomena that control ground displacement be exponential in nature and have values of the order of one half for the thickness factor and one third for the slope factor. No physical theory has been developed to date to confirm these relationships.

Application of Multilinear Regression Model

The following guidance for applying the MLR model is updated from guidance published by Bartlett and Youd (1995). Eq. (6) produces reliable displacement predictions (i.e., plus or minus a factor of two) for input parametric ranges as listed in insert 1 in Fig. 9. The parameter Z_T listed in insert 1 of Fig. 9 is not a statistically significant factor in the MLR model, and hence is not listed in Eq. (6). Z_T is the depth to the top of the layer T_{15} and is included as a limitation to prevent application of Eq. (6) to liquefiable layers deeper than those represented in the case history dataset. Outside the ranges listed in insert 1 of Fig. 9, the response of the equations may be strongly nonlinear and the predicted values uncertain. Thus, caution is warranted when extrapolating Eq. (6) beyond the given limits. Additional guidance for applying the MLR model is given the following commentary.

1. Before applying Eq. (6), the liquefaction susceptibility of the site should be verified through subsurface exploration and liquefaction resistance using evaluated using standard procedures, such as those published by Youd et al. (2000). If the site is nonliquefiable, lateral spread will not occur. Also, liquefiable layers with all SPT $(N_1)_{60}$ values greater than 15 are too dense and dilative for lateral spread to occur;
2. As indicated by the range of magnitudes listed in insert 1 of Fig. 9, the data in the compiled data set are largely from earthquakes with magnitudes between 6 and 8. Extrapolation to magnitudes beyond this range increases the uncertainty of predicted displacements. Because lateral spread displacements are generally small for earthquakes with magnitudes less than 6, Eqs. (4) and (5) may be applied to smaller earthquakes for engineering purposes provided conservative allowance is made for the greater uncertainty. Because of the sparsity of data for earthquakes with magnitudes larger than 8, predicted displacements for these large earthquakes are uncertain. Although, Eq. (6) yields predicted lateral displacements within a factor of two for the few measured displacements reported from the 1964 Alaskan earthquake ($M=9.2$), more case history data are required to fully verify the equations for large earthquakes;
3. In applying the MLR equations to sites with a distant free face, one must decide whether to apply the free-face or ground-slope equation. Predicted displacements using Eq. (6) are usually small for sites at distances greater than 100 times the height of the free face ($W<1\%$). For such small values of W, ground-slope conditions generally control predicted displacement and Eq. (6*b*) should be used. Where there is a question of which equation to use, D_H may be estimated using both Eqs. (6*a*) and (6*b*) and the larger predicted displacement applied. Summing the two predicted displacements is overly conservative; thus, only the larger of the two predicted values need be used. For free-face ratios greater than 5%, free-face conditions generally control the displacement behavior and Eq. (6*a*) should be used. Because the MLR database is comprised mainly of cases with free-face ratios smaller than 20%, caution is warranted when applying Eq. (6*a*) at sites with W greater than 20% (which only occurs very near a free face). Slumping or even flow failure may occur at such localities and generate displacements larger than those predicted by Eq. (6). Similarly, for sites where the ground slope, S, exceeds 6%, Eq. (6*b*) may underpredict displacement due to the possible occurrence of flow failure in contractive soils.
4. The bulk of the data in the case history database are from sites underlain liquefiable layers of well-graded to poorly graded sands, silty sands, sandy silts, and well-graded gravels. A few sites are also underlain by nonplastic silt. Fig. 5 delineates the ranges of grain sizes in the dataset and the applicable range of grain sizes for use with Eqs. (6*a*) and (6*b*); and
5. Eqs. (6*a*) and (6*b*) are appropriate for estimating ground displacement at stiff soil sites in the western U.S. and Japan where attenuation of strong ground motion with distance from the causative fault is relatively high. For other seismic regions (e.g., Eastern United States) or for liquefiable sites underlain by soft soils that may strongly amplify weak ground motions, the equivalent distance term, R_{eq}, is required to account for the stronger motions than would likely occur at stiff western U.S. sites. R_{eq}, is determined from the chart in Fig. 10. To use this chart, the mean a_{max} value expected at the site for the design earthquake is plotted against earthquake magnitude. The equivalent source distance, R_{eq}, is then read from the chart and used in Eqs. (6*a*) or (6*b*). Note that mean acceleration values must be used in this procedure; use of higher values, such as a mean plus one standard deviation, will lead to erroneous and overly conservative results. The curves in Fig. 10 have been updated from those published previously by Bartlett and Youd (1992, 1995). The curves are based on average peak horizontal ground accelerations, a_{max}, calculated from three currently used attenuation relations: (1) Abrahamson and Silva (1997), (2) Boore, Joyner, and Fumal (1997), and (3) Campbell (1997). Notes on values applied in each of these relationships are listed in the caption of Fig. 10.

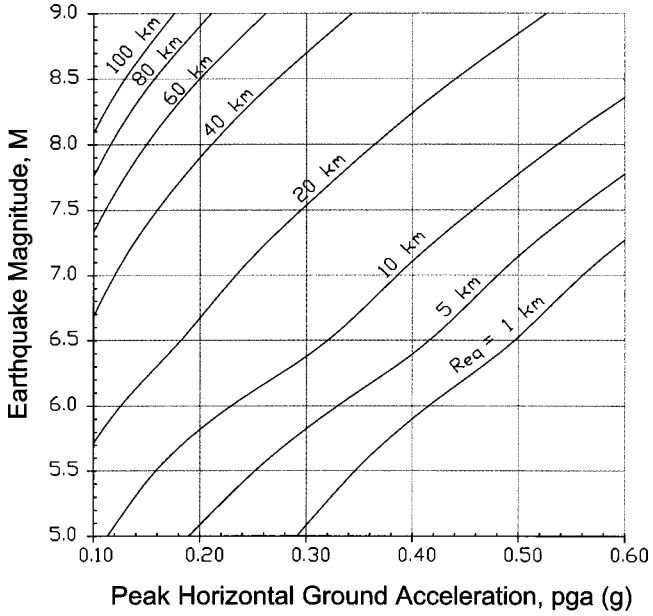

Fig. 10. Graph for determining equivalent source distance, R_{eq}, from magnitude, M, and peak acceleration, a_{max} (revised from Bartlett and Youd 1992, 1995). The above curves are the averages of pga from three different attenuation relations: Abrahamson and Silva (1997); Boore et al. (1997); and Campbell (1997). For the Abrahamson and Silva (1997) relation, the following parameters were used in the regression equation: a) R equals the distance to the fault rupture, b) fault type was set to ''otherwise'', c) HW=hanging wall factor was set to 1, which implies that sites are found on the hanging wall, d) site classification was set to 1 for deep soil sites. For the Boore, Joyner and Fumal (1997) relation, the following parameters were used in the regression equation: a) R is the closest horizontal distance (km) to a vertical projection of fault rupture surface (km), b) Vs in the upper 30 meters was set to 270 m/s, which is the mid range for a medium stiff soil (site class D), c) fault type was set to ''fault mechanism not specified.'' For the Campbell (1997) relation, the following parameters were used in the regression equation: a) R is the closest distance to the seismogenic rupture surface (km), b) fault style factor was set to ''otherwise'', c) soft rock and hard rock site factors were set to ''otherwise'', which implies a stiff soil site.

Summary and Conclusions

1. The set of lateral spread case histories compiled by Bartlett and Youd (1992) has been revised (1) by correcting errors that were made in recording displacements from sites in Noshiro, Japan; (2) by removal of sites where boundary

shear impeded displacement; and (3) by adding sites from three additional earthquakes, many of which are from localities underlain by gravelly soils. The form of the MLR model was also revised (1) to include a logarithmic form for the mean-grain-size term, $D50_{15}$, (to increase the range of grain sizes for which the predictive equations are valid); and (2) to incorporate an additional magnitude-dependent constant, R_o, to the log R term in the model to prevent the calculation of unreasonably large displacements at small seismic source distances. With these revisions made, the case history data set was re-regressed using MLR procedures to develop Eqs. (6*a* and *b*) for the prediction of lateral spread displacements for free-face and ground-slope conditions, respectively. These improved equations are capable of predicting lateral spread displacements within a factor of plus or minus two for sites for seismic, topographic, and soil properties within the ranges listed in inset 1 of Fig. 9. The regression coefficient for Eq. (6) is slightly improved from that of Bartlett and Youd (1995) (83.6% compared to 82.6%), indicating that the performance of the regression model is also slightly improved.

2. Because displacements greater than 6 m listed in the dataset, occurred during only one earthquake and at one site condition (1964 Niigata, Japan earthquake with lateral spread of banks of the Shinano River toward the river channel), predicted displacements greater than 6 m are poorly constrained and uncertain. Thus, predicted displacements greater than 6 m should not be applied in engineering practice; such large predicted displacements do indicate, however, that displacements are likely to be large.
3. The procedure for predicting lateral spread displacement is diagram shown in Fig. 9. We recommend this procedure and the revised model for engineering practice. This procedure supersedes the models of Bartlett and Youd (1992, 1995) and Youd et al. (1999), and is recommended for engineering practice.

Acknowledgment

The College of Engineering and Technology, Brigham Young University, provided funding for this project.

References

Abrahamson, N. A., and Silva, W. J. (1997). ''Empirical response spectral attenuation relations for shallow crustal earthquake.'' *Seismol. Res. Lett.*, 68(1), 94–127.

Ambraseys, N. N. (1988). ''Engineering seismology.'' *Earthquake Eng. Struct. Dyn.*, 17, 1–105.

Andrus, R. D., and Youd, T. L. (1987). ''Subsurface investigation of a liquefaction induced lateral spread, Thousand Springs Valley, Idaho.'' *U.S. Army Corps of Engineers Misc. Paper GL-87-8*, Vicksburg, Miss., 133.

Andrus, R. D., Stokoe, K. H., II, and Roesset, J. M. (1991). ''Liquefaction of gravelly soil at Pence Ranch during the 1983 Borah Peak, Idaho earthquake.'' *Proc., 5th Int. Conf. on Soil Dynamics and Earthquake Engineering*, Karsruhe, Germany, 251–262.

Bartlett, S. F., and Youd, T. L. (1992). ''Empirical analysis of horizontal ground displacement generated by liquefaction-induced lateral spread.'' *Tech. Rep. No. NCEER-92-0021*, National Center for Earthquake Engineering Research, Buffalo, N.Y., 114.

Bartlett, S. F., and Youd, T. L. (1995). ''Empirical prediction of liquefaction-induced lateral spread.'' *J. Geotech. Eng.*, 121(4), 316–329.

Boore, D. M., Joyner, W. B., and Fumal, T. E. (1997). ''Equations for estimating horizontal response spectra and peak acceleration for western north american earthquakes: A summary of recent work.'' *Seismol. Res. Lett.*, 68(1), 128–153.

Boulanger, R. W., Idriss, I. M., and Mejia, L. H. (1995). ''Investigation and evaluation of liquefaction related ground displacements at Moss Landing during the 1989 Loma Prieta earthquake.'' *Rep. No. UCD/CGM-95/02*, Center for Geotechnical Mdlng., Univ. of Calif. at Davis, Davis, Calif.

Campbell, K. (1997). ''Empirical near-source attenuation relationships for horizontal and vertical components of peak ground acceleration, peak velocity, and pseudo-absolute acceleration response spectra.'' *Seismol. Res. Lett.*, 68(1), 154–179.

Hamada, M., Yasuda, S., Isoyama, R., and Emoto, K. (1986). ''Study on liquefaction induced permanent ground displacements.'' *Mngrph.*, Association for the Development of Earthquake Prediction in Japan, Tokyo.

Hamada, H., Isoyama, R., and Wakamatsu, K. (1995). ''The Hyogo-Ken Nambu (Kobe) earthquake, liquefaction, ground displacement and soil condition in Hanshin area.'' *Mngrph.*, Association for Development of Earthquake Prediction in Japan, School of Science and Engineering, Waseda Univ. and Japan Engineering Consultants, Limited.

Mejia, L. H. (1998). ''Liquefaction at moss landing, in the Loma Prieta, California, earthquake of October 17, 1989.'' *Professional Paper 1551-B*, U.S. Geological Survey, Menlo Park, Calif., 129–150.

Youd, T. L. (1977). ''Reconnaissance report of geotechnical observations for the 4, March 1977 Romanian earthquake.'' *Open-file Rep. No. 77-375*, U.S. Geological Survey, Menlo Park, Calif., 22.

Youd, T. L., Hansen, C. M., and Bartlett, S. F. (1999). ''Revised MLR equations for predicting lateral spread displacement.'' *Proc. 7th U.S.–Japan Workshop on Earthquake Resistant Design of Lifeline Facilities and Countermeasures Against Liquefaction, Seattle, Tech. Rep. No. MCEER-99-0019*, Multidisciplinary Center for Earthquake Engineering Research, Buffalo, N.Y., 99–114.

Youd, T. L., Harp, E. L., Keefer, D. K., and Wilson, R. C. (1985). ''The Borah peak, Idaho earthquake of October 29, 1983—liquefaction.'' *Earthquake Spectra*, 2, 71–90.

Youd, T. L., and Hoose, S. N. (1978). ''Historic ground failures in northern California triggered by earthquakes.'' *Professional Paper 993*, U.S. Geological Survey, Menlo Park, Calif., 177.

Youd, T. L., et al. (2000). ''Liquefaction resistance of soils: Summary report from the 1996 NLEER and 1998 NCEER/NSF workshops on evaluation of liquefaction resistance of soils.'' *J. Geotech. Geoenviron. Eng.*, 127(10), 817–833.

Youd, T. L. (2002). Youd, Hansen, and Bartlett database for induced lateral spreads, ⟨http://www.et.byu.edu/cc/ccweb/faculty/youd/data.html⟩.